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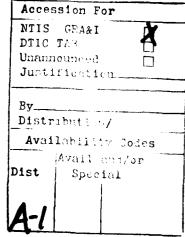
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ABSTRACT

This report describes an effective method for using two-dimensional infinite elements to compute acoustic or magnetic fields in the unbounded fluid region surrounding a submerged vehicle. In this method, finite elements represent the bounded region containing the vehicle and may also be used to represent a layer of fluid surrounding the vehicle. Infinite elements are used to represent the unbounded exterior region. Since infinite elements are not bounded, their shape functions are chosen to contain decay factors to produce convergent integrals. If, from physical or other considerations, the order of decay of the solution as the radius increases is known, infinite elements should be chosen with the same order of decay. The results obtained in this study were found to be within 2 percent when the decay factor of infinite elements matched that of the solution. However, for other problems, the order of decay of the solution may not be known in advance, and, therefore, it may not be possible to match the two rates of decay. For such cases, the errors were found to be as large as 20 percent. In such situations, a layer of finite fluid elements, two elements thick around the structure, reduced the errors to less than 3.5 percent for the modes and decay factors tested.

ADMINISTRATIVE INFORMATION

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INTRODUCTION

To compute magnetic or acoustic fields about submerged vehicles, the vehicle, surrounding water, and interactions between them must be modeled. The field and the structure can each be represented by finite elements and the representations coupled using a method developed by Zienkiewicz and Newton. 1* If the vehicle is submerged in the sea, the extent of the water surrounding it is so great that the region containing the water is best represented as having infinite extent. If finite elements are used to solve such a problem, one can model the structure and a reasonable extent of the finite region of the medium around the structure. This leaves the problem of accounting for the infinite region surrounding the finite

^{*}A complete listing of references is given on page 15.

region modeled with finite elements. Various methods have been proposed for representing the surrounding infinite region. Several examples are:

- The surrounding region can be truncated by modeling a large part of it with finite elements and applying boundary conditions that approximate the effects of the remainder of the infinite region.
- The solution can be expanded as a series of analytic functions in the surrounding region with the coefficients introduced as unknowns.
- The effects of the surrounding region can be represented by an integral equation on the boundary of the region surrounding the finite elements.
- ullet The surrounding region can be modeled with infinite elements. Zienkiewicz et al. 2 compare these methods and find that each has certain advantages and disadvantages.

In this report, infinite elements are used to represent the surrounding infinite region. An infinite element represents a sector of the infinite region extending from the boundary of the finite region. In both finite and infinite elements, the unknown function is approximated by shape functions and a functional involving the shape functions is integrated over the area or volume of the element. To obtain convergent integrals for infinite elements, one of two schemes is used. 3 The first incorporates decay factors in the shape functions that vary in the direction that extends to infinity. The other scheme maps the infinite element into a standard square or cube. This mapping also requires decay factors that compress the infinite element into a finite region. The decay factors may take one of several forms; they may decrease exponentially with the distance r from a fixed point, or decrease as a power of r. If a solution has a component that decreases as 1/r as r tends to infinity, usually that component will dominate for very large values of r. However, components that are significant for moderately large r effect the accuracy obtained with infinite elements. For some problems, the rate of decay of the predominant component of the solution as r tends to infinity, may be deduced a priori from physical or other considerations. For such problems, one could choose a decay factor that matches the behavior of this component as r approaches infinity. It would be expected that this choice would produce the most accurate results. However, the solution may have more significant components than the infinite element has decay factors, or it may have significant components that are not known

a priori. That is, the solution may have significant components that decrease as $1/r^m$ for one or more values of m, where the factors $1/r^m$ are not represented by the decay factors of an infinite element. Thus, the question arises of how well an infinite element will approximate a solution that has components that decrease with orders different from those represented by the decay factors.

This report gives results for the two-dimensional problem of an infinitely long cylindrical shell submerged in an infinite acoustic fluid. The ultimate interest is in three-dimensional problems, but a computational solution using infinite elements for the two-dimensional problem is easier to develop and an exact solution is available for comparison. A two-dimensional problem is quite different in character from a corresponding three-dimensional problem. Rather than simulating the complete three-dimensional solution from the two-dimensional solution, this study examines the performance characteristics of two-dimensional infinite elements and, based on this, conjectures the performance of threedimensional infinite elements. This is done for two configurations and several formulations of the infinite elements. The reported results show that the approximate solutions, produced when only infinite elements are used to model the fluid, have small errors if the decay factors match the known rate of decay of the exact solution. Also, the results show that, for cases when decay factors for the solution and the infinite elements did not match, the computed solutions were found to be quite accurate if a layer of the fluid surrounding the vehicle was modeled by finite elements.

The advantages of using infinite elements are: (1) they maintain symmetry and reasonable bandwidths in the matrices produced, and (2) it is easy to implement the method using available finite element programs. On the other hand, some formulations of infinite elements introduce more degrees of freedom than do competing methods. In some cases extra effort must be made to ensure element shapes that provide unique mappings. So, for some problems, these characteristics may require additional effort in producing the numerical model.

NATURAL FREQUENCIES OF A SUBMERGED CYLINDER

We considered the problem of an infinitely long cylindrical shell submerged in an acoustic fluid. The problem is to determine the natural frequencies of the system consisting of the vibrating structure interacting with the surrounding fluid.

The axis of the cylindrical shell is aligned with the z-axis. The motion of the cylinder's surface and the acoustic pressure are assumed to be independent of the z-coordinate. Since it is assumed that there is no variation of shell displacements or fluid pressure in the z-direction, a two-dimensional problem can be obtained by projecting the cylinder and the region containing fluid into the x,y-plane.

FINITE AND INFINITE ELEMENT MODEL OF A FLUID-STRUCTURE SYSTEM

Natural frequencies of the fluid-structure system are computed using a NASTRAN model containing both finite and infinite elements. Equations associated with the cylindrical shell, the acoustic fluid, and the coupling between them are combined to obtain a matrix equation for the coupled system. For an assumed time harmonic solution, the matrix equation becomes the equation for an eigenvalue problem for determining the natural frequencies of the coupled system. This work extends and modifies the work of Schroeder and Marcus⁴ by introducing infinite elements and using the added mass approximation.

In the two-dimensional finite element model, the cylindrical shell is represented by a ring of one-dimensional bar elements. The fluid region is the region of the x,y-plane outside the ring, and is modeled using two-dimensional membrane finite elements by an analogy for the Laplace operator and two-dimensional infinite elements.

Standard finite element modeling procedures produce the structure matrix equation associated with the $\mbox{ring}^{\,5}$

 $M\ddot{u} + Ku = f$

where M = mass matrix

K = stiffness matrix

u = displacement vector

f = force vector acting on the ring

The method of analogies uses structural membrane elements to produce a matrix equation for the acoustic fluid. The method of analogies 6 is applied by giving Young's modulus E the value unity, and the shear modulus G the value 10^5 . The differential equation for the acoustic pressure P is the wave equation

$$\frac{3^2 P}{3 t^2} - e^{2 \nabla^2 P} = 0$$

where c is the speed of sound in the fluid.

If the acoustic pressure is harmonic in time, that is, $P(\vec{x},t) = p(\vec{x})e^{i\omega}t$ where \vec{x} is a point in the x,y-plane, the wave equation becomes

$$k^2p + \nabla^2p = 0$$

where $k = \omega/c$. Using the method of analogies with the finite element approximation produces the acoustic pressure matrix equation

$$k^2Qp + Hp = 0$$

The structure and acoustic pressure matrix equations are coupled by interactions at the fluid-structure interface. The acoustic pressure acting on the interface produces a force vector (with the convention that forces acting outward from the surface are positive) so the matrix equation for the structure becomes

$$M\ddot{u} + Ku = -Ap$$

where A is a matrix whose entries reflect the area of elements on the interface. See Schroeder and Marcus⁴ for details of the analysis of the fluid-structure coupling. The matrix A may be given by either a consistent or a lumped formulation. Since the boundary in this problem is an arc of a circle, the consistent form is used as it is easy to compute. The entries in the area matrix are

$$A_{i,j} = \int_{S} n_{i}N_{i}F_{j}dS$$

where N_1 = the shape function associated with the i-th structural degree of freedom

S = the fluid-structure interface

 $\ensuremath{\text{n}}_i$ = the component of the unit normal to S in the direction of the i-th translational degree of freedom

 F_j = the shape function associated with the j-th fluid degree of freedom. The motion of the structure also affects the acoustic pressure field in the fluid. The acceleration \ddot{u}_n of the surface of the structure normal to the interface between the fluid and the structure is related to the gradient of the acoustic pressure field by the equation

$$\partial p/\partial n = -\rho \ddot{u}_n$$

where ρ is the density of the fluid. This relation is added as a boundary condition and produces a force-type term in the equation for the acoustic pressure field. Passing to the finite element matrix equation results in the addition of an area matrix term

$$k^2Qp + Hp \approx -\rho A^T \ddot{u}$$

The matrix ${\bf A}$ is the same area matrix that appears in the matrix equation for the structure.

For low frequencies, the added mass approximation can be used. This approximation accounts for only the inertial effects of the surrounding fluid. If the frequency ω is small, then k is small and the first term in the preceding equation is negligible.

Combining the matrix equations for the structure and the acoustic field produces the system of equations

$$M\ddot{u} + Ku + Ap = 0$$

$$-\rho A^{T}\ddot{u} + Hp = 0$$

The system of two equations is reduced to one equation by eliminating p

$$M\ddot{u} + Ku + \rho AH^{-1}A^{T}\ddot{u} = 0$$

In this equation, the term $\rho AH^{-1}A^{T}$ is a coefficient of the second time derivative of the displacement of the structure, and thus acts like a mass added to the mass of the structure and is called the "added mass."

Assuming a solution of the form $u(x,t) = U(x)e^{i\omega t}$ results in a matrix eigenvalue problem that is solved for the natural frequencies of the fluid-structure system

$$\det[\omega^2(M + \rho AH^{-1}A^T) - K] = 0$$

INFINITE ELEMENT FORMULATION

The infinite elements used here are of the decay factor type³; their formulation is an extension of the formulation of finite elements. Each infinite element represents a sector of the infinite region radiating outward from the boundary of the region represented by the finite elements. The difference between an infinite element of this type and a finite element is that the infinite element represents an infinite region and the shape functions associated with the infinite element incorporate decay factors, factors that decrease to zero fast enough to produce convergent integrals over the infinite region.

The formulation of these infinite elements is similar to that of isoparametric finite elements. The region represented by one infinite element is mapped onto a standard infinite strip. Figure 1 shows the mapping and the standard strip. The functions that map the infinite element into the standard strip are

$$\vec{x} = (\vec{x}_1 + \xi \hat{n}_1) L_1(n) + (\vec{x}_2 + \xi \hat{n}_2) L_2(n)$$

where \hat{n}_i are unit vectors normal to the boundary between the finite and infinite elements at the points \vec{x}_i (i = 1,2). The pressure field is approximated by

$$p = p_1H(\xi)L_1(\eta) + p_2H(\xi)L_2(\eta)$$

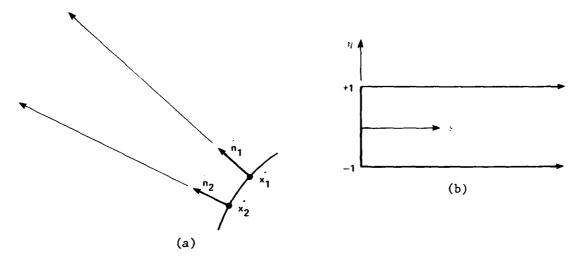


Figure 1 - An Infinite Element (a) and its Parent Strip (b)

where

$$L_1(n) = (1 - n)/2, L_2(n) = (1 + n)/2$$

and

$$H(\xi) = \left(\frac{R}{R + \xi}\right) m$$

for R = R(n), the distance from the origin to the point $\dot{x}(0,n)$ on the boundary of the infinite element. The thape functions incorporate the decay factor H which decrease to zero as ξ becomes infinite to provide for finite integrals. The factor H decreases at the rate $1/r^m$ as r becomes infinite.

At ξ = 0, the factor H takes the value 1 and tends to 0 as ξ becomes infinite. These conditions are necessary, but not sufficient, to produce a good approximation. For example, each of the expressions

$$1/(1+\xi)^m$$
 and $r/(R+\xi^m)$

also satisfies the two preceding requirements, but produces poor results because neither is a good approximation of $1/r^m$ for moderate values of ξ . The numerical integration procedure computes values of the integrand for moderate to large values of ξ , and the shape function H must provide a good approximation for these values.

Care must be taken in forming the infinite elements to avoid a mapping that folds over on itself.³ This may happen if the infinite elements are placed on the fluid-structure interface and a segment of the interface is not convex (as shown in Figure 2).

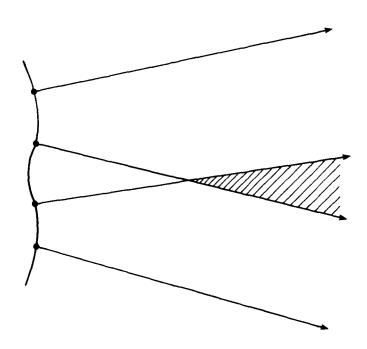


Figure 2 - Nonunique Mapping for Infinite Elements

As shown in Figure 2, the shaded area will be integrated over three times: once by each of the integrations for the center infinite element, and for the elements on either side of the center element. One method of avoiding this nonunique mapping is to make the boundary between the finite element region and the infinite element region convex by inserting finite elements as shown in Figure 3. In this case the nonunique mapping could also be avoided by changing the direction of the vectors $\hat{\mathbf{n}}_1$, but the effect of such a change in the formulation on the performance of the infinite elements would have to be determined.

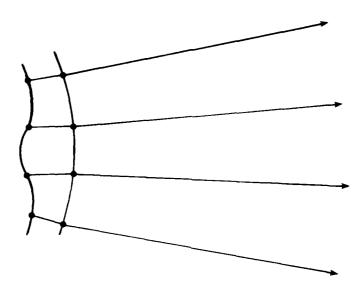


Figure 3 - Use of Finite Elements to Avoid Nonunique Mapping

The coding for infinite elements follows the same steps as for finite elements.³ First, the shape functions and their derivatives are computed, then the Jacobian and the derivatives of the inverse functions are computed, and finally the gradient of the shape functions is integrated numerically over the standard infinite strip.

A modified Gaussian quadrature is used for integration in the infinite direction. The interval $0 < \xi < \infty$ is transformed to -1 < t < 1 by $\xi = (1+t)/(1-t)$, so that

$$\int_{0}^{\infty} f(\xi) d\xi = \int_{-1}^{1} f \frac{1+t}{1-t} \frac{2dt}{(1-t)^{2}}$$

Therefore, for X_i and W_i , the Gauss points and weights for -1 < X < 1, the Gauss points and weights for $0 < \xi < \infty$ are

$$\xi_i = (1+X_i)/(1-X_i)$$
 and $w_i = 2W_i/(1-X_i)^2$

RESULTS OF THE TEST PROBLEMS

The added mass approximation assumes that the pressure in the acoustic field satisfies the potential equation. Therefore, the pressure field in terms of polar coordinates in the infinite exterior region is composed of modes of the form

$$p_n(r,\theta) = (\cos n\theta)/r^n$$

It would be expected that an infinite element would give the best approximation for a mode if the rate of decay of the mode matches that of the element. To investigate how well the method approximates modes whose rates of decay do not match those of the infinite elements, natural frequencies were computed for modes n=2, 3, 4, and 5 using infinite elements with decay rates equal to $1/r^m$ for m=1, 2, 3, and 4. The frequencies computed were compared with frequencies calculated using an analytic solution for the problem. Results from these computations are given in Figures 4 and 5.

The solution of the problem of computing natural vibration modes and frequencies for the submerged cylindrical shell, modeled with finite and infinite elements, was implemented using the NASTRAN program. The actual model for the problem represented one quadrant of the plane. Symmetric and antisymmetric boundary conditions were used to obtain symmetric and antisymmetric vibration modes. To determine how well the solution had converged with refinement of the finite element grid in the azimuthal direction, the problem was solved using 16 and 32 sectors per quadrant. For these cases, two rings of finite elements were used between the structural elements and the infinite elements. The relative errors plotted in Figure 4 are less than 3 percent and decrease with the refinement of the grid, although not greatly. Since higher modes require greater detail in modeling, the errors become larger for these modes. The results show that the solution is converged with respect to azimuthal refinement well enough that futher refinement will not significantly decrease the error. It can also be seen that, although the errors are smallest for the modes whose rate of decay matches the rate of decay of the infinite elements, that is if n = m, the errors are not extremely sensitive to the rate of decay.

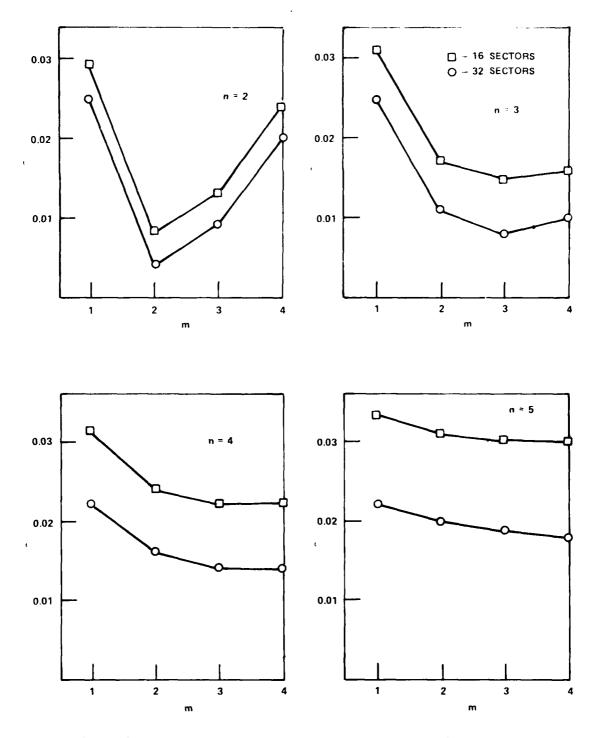


Figure 4 - Comparison of Relative Errors for 16 and 32 Sectors per Quadrant, Using Two Rings of Fluid Finite Elements

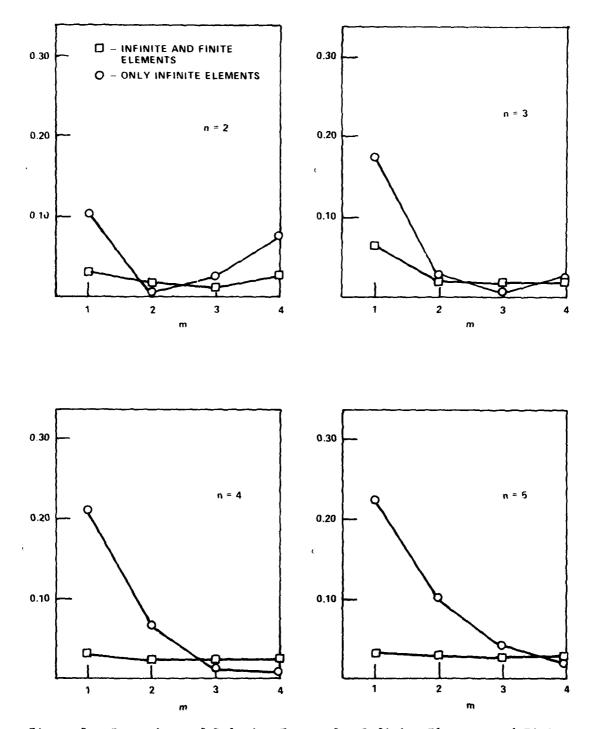


Figure 5 - Comparison of Relative Errors for Infinite Element, and Finite and Infinite Element Solutions

In the previous cases, two rings of fluid finite elements were used between the structural elements and the infinite elements. The problem can also be solved using only infinite elements to represent the fluid. It is useful to learn the effect on the accuracy of the frequencies of the various modes when only infinite elements are used to model the fluid. For these tests, 16 sectors of elements were used in the quadrant. The infinite elements were coupled to the structure at the interface using the same coupling technique that was used with finite elements. Comparisons of solutions obtained using only infinite elements with solutions obtained using both finite and infinite elements are shown in Figure 5. The results show that there is more sensitivity to the rate of decay when only infinite elements are used. The errors are small for modes whose rate of decay matches the rate of decay of the infinite element, but become significant for modes whose rate of decay does not match that of the infinite element.

DISCUSSION OF RESULTS

The results obtained in this study indicate that for fluid-structure interaction problems involving structures submerged in large regions of acoustic fluid, one can expect that the use of infinite fluid elements will give good results. Also, if results for a broad band of frequencies are required, the use of one or more layers of finite elements between the structure and the infinite elements will produce good results; however, additional degrees of freedom will be introduced for these elements. There is also the possibility of using infinite elements that have shape functions incorporating several rates of decay. These elements also introduce additional degrees of freedom. When infinite elements are used in one or a combination of these configurations, one can expect that solutions with satisfactory errors will be obtained.

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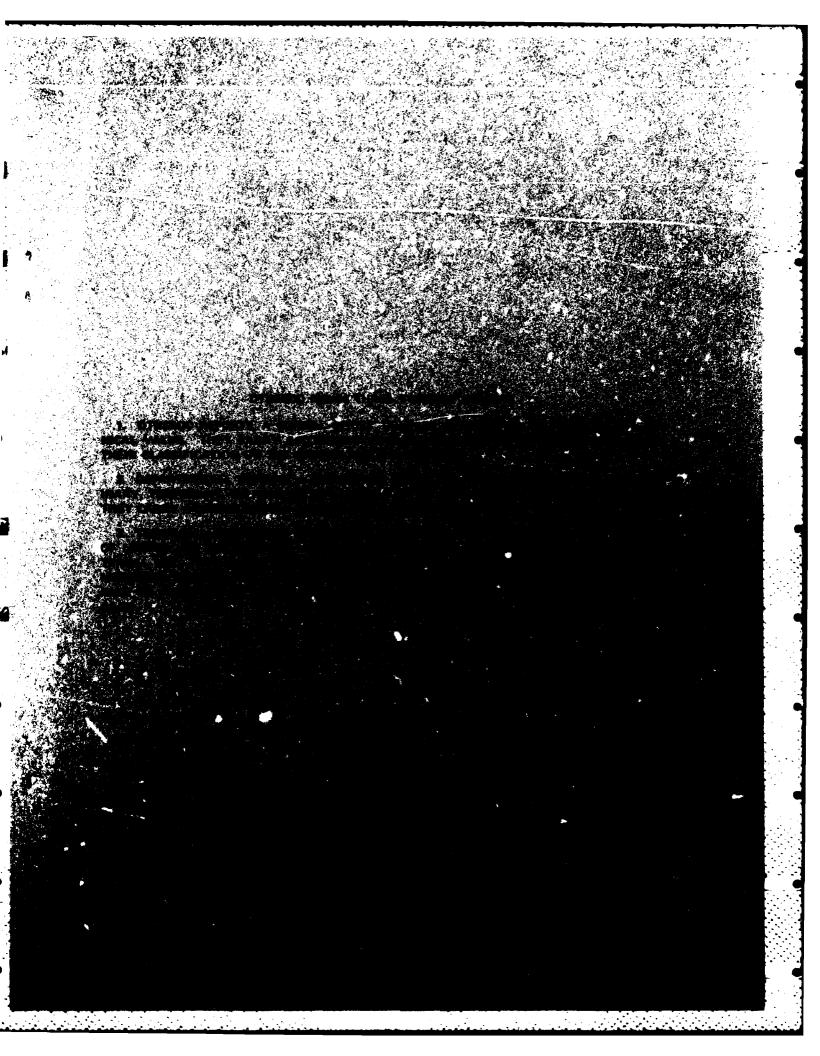
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